APPENDIX B
RATIO EQUATION INSTEAD OF THE RULE OF THREE

From a longitudinal study on percentage calculation in mathematics lessons, it is known that learners of all school types solve a relatively large number of tasks, but achieve less than 50 % correct solutions nationwide.[1] In many schools, the solution scheme "rule of three" is taught to solve percentage problems. The assignment to the terms basic value, proportionality factor causes difficulties. A change from the solution scheme "rule of three" to the solution scheme "ratio equation" does not seem to be possible for the time being. The solution pattern "ratio equation as fraction equation" is not very popular among didacticians.[2] In the opinion of the author of this article, the rule of three has many disadvantages in terms of learning comprehension and application in other subjects. Safe use of the rule of three requires a good understanding of direct and indirect proportionality. It seems that the rule of three is more suitable for simple tasks.

EXEMPLE 1:

400 athletes participated in a road marathon. 35 athletes had to give up before reaching the finish line. What percentage of the athletes who started reached the finish line?

2 Solutions correct

Solution 1

Step 1 Calculation: What percentage of the athletes gave up before.
Step 2: Subtract the result obtained in step 1 from 100 %. The difference is the result you are looking for.

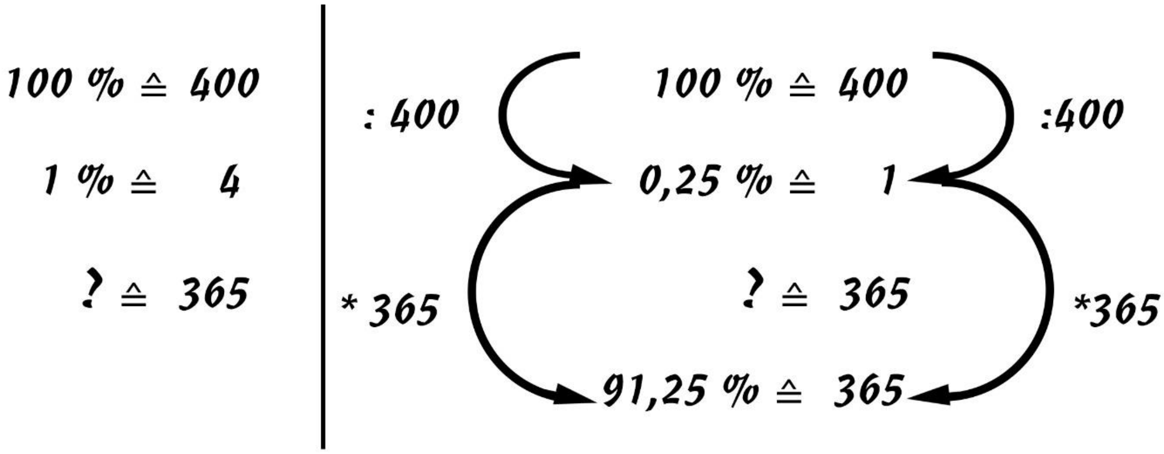
Solution 2 Calculate the percentage in a direct way

Step 1: Calculate how many athletes finished:
400 - 35 = 365 (athletes who arrived at the finish).
Step 2: Calculate what percentage the 365 athletes have of the total number of 400 athletes participating, which corresponds to the result we are looking for

It is possible that the students will find the solution immediately or after thinking about it for a while, as shown in diagram 1 on the right.

Mentally, a lot of information has to be processed at the same time when using the rule of three. The intermediate step with the normalisation to the size "1" obscures the view for correlations and typical patterns.

[1] Hafner, T. (2012). Proportionalität und Prozentrechnung in der Sekundarstufe I. Empirische Untersuchung und didaktische Analysen (Dissertation). Wiesbaden: Vieweg+Teubner
[2] Gudladt, P. (2021). Didaktische Überlegungen. In: Inhaltliche Zugänge zu Anteilsvergleichen im Kontext des Prozentbegriffs. Perspektiven der Mathematikdidaktik. Springer Spektrum, Wiesbaden. https://doi.org/10.1007/978-3-658-32447-6_2



Graphic 1 Solution scheme rule of three

The solution scheme ratio equation is an exact mathematical equation. The translation of the task text into the solution scheme ratio equation is simple, clear and can be mastered mentally without much effort. The student does not need to know anything about proportionality, basic value and proportionality factor.

In other words: Which solution scheme is easier to program with a computer language?

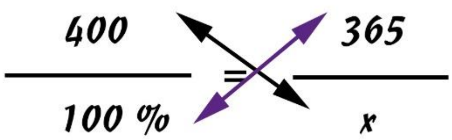
In the beginning, when routine is still lacking, one can mentally link the fraction line with content, and move step by step to the pure mathematically formulated ratio equation. With increasing experience, these steps are automatically omitted.

$$\frac{\text{400 Starter}}{\text{100 Percent}} \text{ Marathon run} = \frac{\text{(400 -35) Athletes at the finish}}{\text{x Percent}} \text{ Marathon run}$$

$$\frac{\text{400 Starter}}{\text{100 Percent}} \text{ Marathon run} = \frac{\text{365 Athletes at the finish}}{\text{x Percent}} \text{ Marathon run}$$

Now the previous equation is rewritten with textual explanations in a pure mathematical equation.

$$\frac{400}{100\ \%} = \frac{365}{x}$$



The resolution crosswise to x is done in the twinkling of an eye. It does not matter whether the unknown quantity is in the numerator or denominator, as long as the ratio equation is correct in content. The resolution by the unknown should be explained step by step for the students. Then you will see much better that the immediate resolution according to the unknown variable x crosswise is a shortcut:

$$x = \frac{365 * 100\ \%}{400}$$

$$x = 91{,}25\ \%$$

Doing without the rule of three means saving time for learning. The pupils do not have to learn a new solution scheme "ratio equation" in addition to the rule of three in order to be able to solve tasks in the natural sciences, for example stoichiometric calculation in chemistry. Lesson time is gained. The advantage of the ratio equation is the simple handling of the underlying pattern, the universal validity and the immediate applicability and transferability to scientific tasks.

A study on chemistry teaching reports that students who proceeded algorithmically were more successful in stoichiometric arithmetic.[3]

EXEMPLE 2:

In a lecture manuscript[4] task variants of the rule of three are discussed in detail (see next graphic). You can see in the tables that the ratio equation is ultimately used in the 3rd row. Why is the ratio equation not introduced in lessons right away?

---

3 Tepner, Oliver. *Effektivität von Aufgaben im Chemieunterricht der Sekundarstufe I*. Vol. 76. Logos Verlag Berlin GmbH, 2008.

4

1.a Jedes Kind beim Klassenfest bekommt ein Glas Apfelsaft.
Für 40 Kinder braucht man 10 l Apfelsaft. Wie viel Apfelsaft braucht man für 32 Kinder?

Dreisatz:

Für 40 Kinder braucht man 10 Liter Apfelsaft.	Bedingungssatz.
Für 32 Kinder braucht man wie viel Apfelsaft?	Fragesatz.

Die drei Sätze des Dreisatzes, die auch so gesprochen werden können:

Für 40 Kinder braucht man 10 Liter Apfelsaft.	Bedingungssatz.
Für 1 Kind braucht man den 40. Teil, also 10 : 40 = 0,25 Liter Apfelsaft.	Schluss auf die Einheit
Für 32 Kinder braucht man 32 mal soviel, also 10:40 · 32 = 8 Liter Apfelsaft.	Schluss auf die Vielheit

Ausführliches Schema, Sätze während des Aufschreibens dazu sprechen:

Zahl der Kinder	Apfelsaft in Litern
40	10
1	$\frac{10}{40}$
32	$\frac{10 \cdot 32}{40} = 8$

„Apfelsaft" gesucht, also *„Apfelsaft"* in die rechte Spalte schreiben

Abgekürztes Schema

Kinder	Apfelsaft in l
40	
1	$\frac{10 \cdot 32}{40} = 8$
32	

Tabelle:

	Zahl der Kinder	Apfelsaft in Litern	
	40	10	
:40	1	$\frac{10}{40}$	:40
·32	32	$\frac{10 \cdot 32}{40} = 8$	·32

„Apfelsaft" gesucht, also *„Apfelsaft"* in die rechte Spalte schreiben

Die Lösung über eine Tabelle bietet sich an, wenn Zuordnungen und ihre Eigenschaften im Vordergrund stehen.

Verhältnisgleichung:

Lösen über Verhältnisgleichung, unbekannte Größe (Variable) möglichst in den Zähler setzen.

Apfelsaft	0 l	x l	10 l
Kinder	0	32	40

$$\frac{x}{32} = \frac{10}{40}$$

oder auch

$$\frac{x}{10} = \frac{32}{40}$$

$\Big\}$ $x = 8$

Graphic 2 Explanation of the rule of three, lecture manuscript on the didactics of mathematics, in German

The hint given below for the ratio equation (see graphic above) to put the unknown in the numerator leads to irritation and possibly unsettles the students. It is also possible that the hint is counterproductive for thinking, because the hint to put the variable in the numerator hinders the student's flow of thought. This is because the sentences formulated for the task already visually contain the solution scheme "ratio equation" (cf. right-hand page below). All that remains is to extract the superfluous text, insert a fraction line between the two numbers and the ratio equation is on the paper and can be calculated effortlessly.

Task text in clearly arranged notation

Task in German

Für 40 Kinder braucht man 10 Liter Apfelsaft.
Für 32 Kinder braucht man wie viel Apfelsaft?

o

Task in English

For 40 children you need 10 litres apple juice?
For 32 children you need how much apple juice?

Ratio equation to the task text

$$\frac{40}{32} = \frac{10 \text{ litres}}{x \text{ litres}}$$

or

$$\frac{40}{10 \text{ litres}} = \frac{32}{x \text{ litres}}$$

If the pupils are shown that the following ratio equations also result from the equation that has now been formulated, the pupils' acceptance of calculating by means of ratio equations will be great, because the risk of making mistakes is very low.

$$\frac{40}{32} = \frac{10 \text{ litres}}{x \text{ litres}} \qquad \frac{40}{10 \text{ litres}} = \frac{32}{x \text{ litres}}$$

It is in everyone's interest that the level of performance in the error-free calculation of percentage and ratio tasks in Germany improves strongly, from approx. 50 % correct solutions of the pupils across all school types towards 100 % correct solutions.